

On the CFT duals for near-extremal black holes

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Abstract

We consider Kerr-Newman-AdS-dS black holes near extremality and work out the near-horizon geometry of these near-extremal black holes. We identify the exact $U(1)_L \times U(1)_R$ isometries of the near-horizon geometry and provide boundary conditions enhancing them to a pair of commuting Virasoro algebras. The conserved charges of the corresponding asymptotic symmetries are found to be well defined and non-vanishing and to yield central charges $c_L \neq 0$ and $c_R = 0$. The Cardy formula subsequently reproduces the Bekenstein-Hawking entropy of the black hole. This suggests that the near-extremal Kerr-Newman-AdS-dS black hole is holographically dual to a non-chiral two-dimensional conformal field theory.

1 Introduction

In the seminal work [1], quantum gravity on three-dimensional anti-de Sitter (AdS) space was found to be holographically dual to a two-dimensional conformal field theory (CFT). The Kerr/CFT correspondence [2] extends this duality by asserting that quantum theory on the near-horizon geometry of the extremal four-dimensional Kerr black hole [3] is holographically dual to a chiral CFT in two dimensions. This near-horizon geometry has a $U(1)_L \times SL(2, \mathbb{R})_R$ isometry group and is called the NHEK geometry, and using the formalism of [4], it was found [2] that certain boundary conditions enhance the $U(1)_L$ symmetry to a Virasoro algebra. The consistency of these boundary conditions was subsequently confirmed in [5, 6], while a partial classification of boundary conditions with the same properties was provided in [7]. Boundary conditions enhancing the $SL(2, \mathbb{R})_R$ isometries to a Virasoro algebra were examined in [8, 7]. Strong evidence [2] for the Kerr/CFT correspondence is found in the exact agreement between the macroscopic Bekenstein-Hawking entropy [9] of the black hole and the Cardy formula for the dual CFT entropy. This analysis has been successfully generalized and applied to a variety of extremal black holes [10, 11, 12, 13, 5, 6, 14]. The extension to the near-horizon geometry of the extremal Kerr-Newman-AdS-dS black hole [15, 16], in particular, is worked out in [16].

Extending the Kerr/CFT correspondence to the *near*-extremal Kerr black hole, however, has presented some serious challenges. The dual two-dimensional CFT should be non-chiral and hence accommodate two mutually commutative Virasoro algebras. Based on a variety of approaches, several insightful results on these near-extremal black holes have been obtained [17]. Extending the approach of [18], in particular, the work [19] on black-hole superradiance and the subsequent generalizations thereof [20] have provided highly non-trivial support for the Kerr/CFT correspondence. These results, however, are all based on deviations from the geometrical approach of [2] since consistent boundary conditions which allow for both left- and right-moving sectors have been identified only very recently [21]. Another very recent development is the analysis of the so-called hidden conformal symmetry [22] of the Kerr black hole. It is noted that this symmetry is not a symmetry of the spacetime geometry.

It is the main objective of the present work to extend the work [21] on the near-horizon geometry of the near-extremal Kerr black hole, the so-called near-NHEK geometry, to the near-horizon geometry of the near-extremal Kerr-Newman-AdS-dS black hole using the results [16] on the extremal case. We work out the metric of this near-horizon geometry, identify its exact $U(1)_L \times U(1)_R$ isometry group, and provide boundary conditions enhancing these symmetries to a pair of commuting Virasoro algebras. The conserved charges of the corresponding asymptotic symmetries are found to be well defined and non-vanishing and to yield central charges $c_L \neq 0$ and $c_R = 0$. The Cardy formula subsequently reproduces the Bekenstein-Hawking entropy of the black hole. This supports the assertion that the near-extremal Kerr-Newman-AdS-dS black hole is holographically dual to a *non-chiral* two-dimensional CFT. Here we only consider the geometry of the Kerr-Newman-AdS-dS black hole but hope to address elsewhere the inclusion of the gauge field.

It is noted that the Virasoro algebra has been observed in other black-hole contexts as well. Extending the work [23] on the entropy of three-dimensional black holes such as the BTZ black hole [24], it was found in [25, 26] that a copy of the Virasoro algebra appears in the near-horizon region of any black hole and that it reproduces the black-hole entropy using the Cardy formula.

2 Kerr-Newman-AdS-dS black hole

2.1 Geometry

In Boyer-Lindquist-type coordinates, and using the unit convention where $G = \hbar = c = 1$, a Kerr-Newman-AdS-dS black hole [15, 16] is described by

$$ds^2 = -\frac{\Delta_r}{\rho^2} \left(dt - \frac{a}{\Xi} \sin^2 \theta d\hat{\phi} \right)^2 + \frac{\rho^2}{\Delta_r} d\hat{r}^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 + \frac{\Delta_\theta}{\rho^2} \sin^2 \theta \left(a d\hat{t} - \frac{\hat{r}^2 + a^2}{\Xi} d\hat{\phi} \right)^2 \quad (2.1)$$

where

$$\begin{aligned} \Delta_r &= (\hat{r}^2 + a^2) \left(1 + \frac{\hat{r}^2}{\ell^2} \right) - 2M\hat{r} + q^2, & q^2 &= q_e^2 + q_m^2 \\ \Delta_\theta &= 1 - \frac{a^2}{\ell^2} \cos^2 \theta, & \rho^2 &= \hat{r}^2 + a^2 \cos^2 \theta, & \Xi &= 1 - \frac{a^2}{\ell^2} \end{aligned} \quad (2.2)$$

This metric is parameterized by the ADM mass $M_{ADM} = M/\Xi^2$, angular momentum $J = aM/\Xi^2$ and electric and magnetic charges $Q_e = q_e/\Xi$ and $Q_m = q_m/\Xi$, and it is AdS (dS) for positive (negative) renormalized cosmological constant ℓ^{-2} . It reduces to Kerr-Newman for $\ell^{-2} = 0$ and further to Kerr for $q^2 = 0$. The horizons are located at the positive zeros of Δ_r , and the value of \hat{r} at the outer horizon is denoted by r_+ . The Hawking temperature T_H , angular velocity Ω_H of the event horizon, and entropy S are given by

$$T_H = \frac{r_+ \left(1 + \frac{a^2}{\ell^2} + \frac{3r_+^2}{\ell^2} - \frac{a^2 + q^2}{r_+^2} \right)}{4\pi(r_+^2 + a^2)}, \quad \Omega_H = \frac{a\Xi}{r_+^2 + a^2}, \quad S = \frac{\pi(r_+^2 + a^2)}{\Xi} \quad (2.3)$$

For later convenience, we follow [16] and introduce

$$r_0^2 = \frac{(r_+^2 + a^2) \left(1 - \frac{r_+^2}{\ell^2} \right)}{1 + \frac{6r_+^2}{\ell^2} - \frac{3r_+^4}{\ell^4} - \frac{q^2}{\ell^2}}, \quad \rho_+^2 = r_+^2 + a^2 \cos^2 \theta \quad (2.4)$$

and

$$\Gamma(\theta) = \frac{\rho_+^2 r_0^2}{r_+^2 + a^2}, \quad \alpha(\theta) = \frac{r_+^2 + a^2}{\Delta_\theta r_0^2}, \quad \gamma(\theta) = \frac{\Delta_\theta \sin^2 \theta (r_+^2 + a^2)^2}{\Xi^2 \rho_+^2}, \quad k = \frac{2a\Xi r_0^2 r_+}{(r_+^2 + a^2)^2} \quad (2.5)$$

2.2 Extremal black holes

When the inner and outer horizons coalesce, the black hole is said to be extremal. This happens when the otherwise single pole r_+ of Δ_r is a double pole in which case

$$\Delta_r(r_+) = \Delta'_r(r_+) = 0 \quad (2.6)$$

We denote the value of \hat{r} at this single horizon by \bar{r} and use the related notation \bar{a} , \bar{T}_H and $\bar{\Omega}_H$, in particular, for the evaluation of the various parameters at extremality. The conditions (2.6) can be used to express M and \bar{a}^2 in terms of \bar{r}

$$M = \frac{\bar{r} \left[\left(1 + \frac{\bar{r}^2}{\ell^2} \right)^2 - \frac{q^2}{\ell^2} \right]}{1 - \frac{\bar{r}^2}{\ell^2}}, \quad \bar{a}^2 = \frac{\bar{r}^2 \left(1 + \frac{3\bar{r}^2}{\ell^2} \right) - q^2}{1 - \frac{\bar{r}^2}{\ell^2}} \quad (2.7)$$

and we readily recover $\bar{T}_H = 0$.

2.3 Near-horizon geometry of extremal black holes

To describe the near-horizon geometry of the extremal Kerr-Newman-AdS-dS black hole, we follow [3, 16] and introduce the new coordinates t, r, ϕ where

$$\hat{t} = \frac{t\bar{r}_0}{\epsilon}, \quad \hat{r} = \bar{r} + \epsilon\bar{r}_0 r, \quad \hat{\phi} = \phi + \frac{\bar{\Omega}_H \bar{r}_0 t}{\epsilon} \quad (2.8)$$

The near-horizon geometry is obtained by taking the limit $\epsilon \rightarrow 0$ in which case the metric (2.1) becomes

$$d\bar{s}^2 = \bar{\Gamma}(\theta) \left(-r^2 dt^2 + \frac{dr^2}{r^2} + \bar{\alpha}(\theta) d\theta^2 \right) + \bar{\gamma}(\theta) (d\phi + \bar{k} r dt)^2 \quad (2.9)$$

The exact isometry group of the near-horizon geometry (2.9) is $U(1)_L \times SL(2, \mathbb{R})_R$ and it is generated by

$$\{\partial_\phi\} \cup \left\{ \partial_t, t\partial_t - r\partial_r, \left(t^2 + \frac{1}{r^2}\right)\partial_t - 2tr\partial_r - \frac{2\bar{k}}{r}\partial_\phi \right\} \quad (2.10)$$

2.4 Near-extremal geometry

We are interested in infinitesimal excitations above extremality of the Kerr-Newman-AdS-dS black hole. To describe this near-extremal geometry, we consider the situation where the outer horizon r_+ is infinitesimally larger than the extremal horizon \bar{r}

$$r_+ = \bar{r} + \epsilon\bar{\kappa} \quad (2.11)$$

Here $\bar{\kappa}$ is expressed in terms of the parameters of the extremal black hole, while ϵ is an infinitesimal parameter whose limit $\epsilon \rightarrow 0$ we are ultimately interested in, where

$$a^2 = \bar{a}^2 - \frac{\epsilon^2 \bar{\kappa}^2 (\bar{r}^2 + \bar{a}^2)}{\bar{r}_0^2 (1 + \frac{\bar{r}^2}{\bar{\ell}^2})} + \mathcal{O}(\epsilon^3), \quad T_H = \frac{\epsilon\bar{\kappa}}{2\pi\bar{r}_0^2} + \mathcal{O}(\epsilon^2), \quad \Omega_H = \bar{\Omega}_H \left(1 - \frac{2\epsilon\bar{\kappa}\bar{r}}{\bar{r}^2 + \bar{a}^2}\right) + \mathcal{O}(\epsilon^2) \quad (2.12)$$

Following [11], we introduce

$$T_L = \lim_{r_+ \rightarrow \bar{r}} \frac{T_H}{\bar{\Omega}_H - \Omega_H} = \frac{1}{2\pi\bar{k}} \quad (2.13)$$

and interpret it as the Frolov-Thorne temperature [27] associated with the azimuthal angle ϕ . Although the asymptotically-defined Hawking temperature T_H vanishes in the limit $\epsilon \rightarrow 0$, we can use that T_H/ϵ is non-vanishing and finite to introduce

$$T_R = \lim_{r_+ \rightarrow \bar{r}} \frac{r_0^2 T_H}{r_+ \epsilon} = \frac{\bar{\kappa}}{2\pi\bar{r}} \quad (2.14)$$

and interpret it as a temperature associated with the near-extremality of the geometry.

2.5 Near-horizon geometry of near-extremal black holes

To describe the near-horizon geometry of the near-extremal Kerr-Newman-AdS-dS black hole, we use the coordinates t, r, ϕ where

$$\hat{t} = \frac{\bar{r}_0^2 t}{\epsilon}, \quad \hat{r} = \bar{r} + \epsilon(r + \lambda\bar{\kappa}), \quad \hat{\phi} = \phi + \frac{\bar{\Omega}_H \bar{r}_0^2 t}{\epsilon} \quad (2.15)$$

and where we have introduced the additional parameter λ . In the limit $\epsilon \rightarrow 0$, we find

$$\Delta_r = \frac{\epsilon^2(\bar{r}^2 + \bar{a}^2)}{\bar{r}_0^2} (r + (\lambda - 1)\bar{\kappa})(r + (\lambda + 1)\bar{\kappa}) + \mathcal{O}(\epsilon^3) \quad (2.16)$$

and hence

$$\begin{aligned} ds^2 &= \Gamma(\theta) \left[- (r + (\lambda - 1)\bar{\kappa})(r + (\lambda + 1)\bar{\kappa}) dt^2 + \frac{dr^2}{(r + (\lambda - 1)\bar{\kappa})(r + (\lambda + 1)\bar{\kappa})} + \alpha(\theta) d\theta^2 \right] \\ &+ \gamma(\theta) [d\phi + k(r + \lambda\bar{\kappa}) dt]^2 \end{aligned} \quad (2.17)$$

For all λ , this is invariant under the action of the exact isometry group $U(1)_L \times U(1)_R$ generated by

$$\{\partial_\phi\} \cup \{\partial_t\} \quad (2.18)$$

These isometries commute with translations in r generated by ∂_r and realized by varying λ . We note that the determinant of the metric (2.17) only depends on θ as

$$\sqrt{-g} = \sqrt{\Gamma^3(\theta)\alpha(\theta)\gamma(\theta)} = \frac{\sin\theta\rho_+^2 r_0^2}{\Xi} \quad (2.19)$$

In the following, focus will be on $\lambda = 1$ in which case

$$r = \frac{\hat{r} - r_+}{\epsilon} \quad (2.20)$$

and

$$ds^2|_{\lambda=1} = \Gamma(\theta) \left(-r(r + 2\bar{\kappa}) dt^2 + \frac{dr^2}{r(r + 2\bar{\kappa})} + \alpha(\theta) d\theta^2 \right) + \gamma(\theta) (d\phi + k(r + \bar{\kappa}) dt)^2 \quad (2.21)$$

As above, special cases are obtained by setting $\ell^{-2} = 0$ or $q^2 = 0$. The near-NHEK geometry [13, 5, 19, 21], in particular, follows when both of these conditions apply. We note that the coordinate convention for the near-NHEK geometry used in [19, 21] differs from the one following from (2.15), but is obtained by the replacement $t, r \rightarrow t/M, Mr$.

3 Holographically dual CFT

3.1 Boundary conditions and conserved charges

We are interested in perturbations $h_{\mu\nu}$ of the near-horizon geometry (2.21) of the near-extremal Kerr-Newman-AdS-dS black hole. Following [21] on the near-NHEK geometry, we introduce the fall-off conditions

$$h_{\mu\nu} = \mathcal{O} \begin{pmatrix} r^2 & r^{-3} & r^1 & r^{-2} \\ & r^{-4} & r^{-1} & r^{-3} \\ & & 1 & r^{-2} \\ & & & r^{-2} \end{pmatrix}, \quad h_{\mu\nu} = h_{\nu\mu} \quad (3.1)$$

in addition to a couple of supplementary conditions to be discussed below. Asymptotic symmetries are generated by the diffeomorphisms whose action on the metric generates metric fluctuations

compatible with the full set of boundary conditions. Initially focusing on the fall-off conditions (3.1), we find the asymptotic Killing vectors

$$\begin{aligned} K_\epsilon &= [\mathcal{O}(r^{-4})]\partial_t + [-(r+\bar{\kappa})\epsilon'(\phi) + \mathcal{O}(r^{-1})]\partial_r + [\epsilon(\phi) + \mathcal{O}(r^{-3})]\partial_\phi + [\mathcal{O}(r^{-2})]\partial_\theta \\ \mathcal{K}_\varepsilon &= [\varepsilon(t) + \mathcal{O}(r^{-4})]\partial_t + [\mathcal{O}(r^{-1})]\partial_r + [\mathcal{O}(r^{-3})]\partial_\phi + [\mathcal{O}(r^{-2})]\partial_\theta \end{aligned} \quad (3.2)$$

where $\epsilon(\phi)$ and $\varepsilon(t)$ are smooth functions. The generators of the corresponding asymptotic symmetries thus read

$$\xi = -(r+\bar{\kappa})\epsilon'(\phi)\partial_r + \epsilon(\phi)\partial_\phi, \quad \zeta = \varepsilon(t)\partial_t \quad (3.3)$$

and form a commuting pair of centreless Virasoro algebras

$$[\xi_\epsilon, \xi_{\hat{\epsilon}}] = \xi_{\epsilon\hat{\epsilon}' - \epsilon'\hat{\epsilon}}, \quad [\zeta_\varepsilon, \zeta_{\hat{\varepsilon}}] = \zeta_{\varepsilon\hat{\varepsilon}' - \varepsilon'\hat{\varepsilon}}, \quad [\xi_\epsilon, \zeta_\varepsilon] = 0 \quad (3.4)$$

Along ξ and ζ , the Lie derivatives of the near-horizon geometry (2.21) are worked out to be

$$\begin{aligned} \mathcal{L}_\xi g_{\mu\nu} &= \begin{pmatrix} 2(\Gamma - k^2\gamma)(r+\bar{\kappa})^2\epsilon'(\phi) & 0 & 0 & 0 \\ 0 & \frac{2\bar{\kappa}^2\Gamma\epsilon'(\phi)}{r^2(r+2\bar{\kappa})^2} & -\frac{\Gamma(r+\bar{\kappa})\epsilon''(\phi)}{r(r+2\bar{\kappa})} & 0 \\ 0 & -\frac{\Gamma(r+\bar{\kappa})\epsilon''(\phi)}{r(r+2\bar{\kappa})} & 2\gamma\epsilon'(\phi) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ \mathcal{L}_\zeta g_{\mu\nu} &= \epsilon'(t) \begin{pmatrix} -2\Gamma r(r+2\bar{\kappa}) + 2k^2\gamma(r+\bar{\kappa})^2 & 0 & k\gamma(r+\bar{\kappa}) & 0 \\ 0 & 0 & 0 & 0 \\ k\gamma(r+\bar{\kappa}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (3.5)$$

where we have introduced the abbreviations

$$\Gamma = \Gamma(\theta), \quad \alpha = \alpha(\theta), \quad \gamma = \gamma(\theta) \quad (3.6)$$

To an asymptotic symmetry generator η , one associates [4] the conserved charge

$$Q_\eta = \frac{1}{8\pi} \int_{\partial\Sigma} \sqrt{-g} k_\eta[h; g] = \frac{1}{8\pi} \int_{\partial\Sigma} \frac{\sqrt{-g}}{4} \epsilon_{\alpha\beta\mu\nu} d_\eta^{\mu\nu}[h; g] dx^\alpha \wedge dx^\beta \quad (3.7)$$

where

$$d_\eta^{\mu\nu}[h; g] = \eta^\nu D^\mu h - \eta^\nu D_\sigma h^{\mu\sigma} + \eta_\sigma D^\nu h^{\mu\sigma} - h^{\nu\sigma} D_\sigma \eta^\mu + \frac{1}{2} h D^\nu \eta^\mu + \frac{1}{2} h^{\sigma\nu} (D^\mu \eta_\sigma + D_\sigma \eta^\mu) \quad (3.8)$$

and where $\partial\Sigma$ is the boundary of a three-dimensional spatial volume near spatial infinity. Indices are lowered and raised using the background metric $g_{\mu\nu}$ and its inverse, D_μ denotes a background covariant derivative, while h is defined as $h = g^{\mu\nu} h_{\mu\nu}$. To be a well-defined charge in the asymptotic limit, the underlying integral must be finite as $r \rightarrow \infty$. If the charge vanishes, the asymptotic symmetry is rendered trivial. The asymptotic symmetry group is then generated by the diffeomorphisms whose charges are well-defined and non-vanishing. The algebra generated by the set of well-defined charges is governed by the Dirac brackets computed [4] as

$$\{Q_\eta, Q_{\hat{\eta}}\} = Q_{[\eta, \hat{\eta}]} + \frac{1}{8\pi} \int_{\partial\Sigma} \sqrt{-g} k_\eta[\mathcal{L}_{\hat{\eta}} g; g] \quad (3.9)$$

where the integral yields the eventual central extension.

Now, considering the asymptotic r -expansion of $k_{\partial_t}[h; g]$, we find that its divergent term (linear in r) is independent of $\bar{\kappa}$ and that its constant term (independent of r) is at most linear in $\bar{\kappa}$. Subleading terms in r can be ignored as $r \rightarrow \infty$. We thus follow [21] and impose the condition

$$(1 - \bar{\kappa}\partial_{\bar{\kappa}})Q_{\partial_t} = 0 \quad (3.10)$$

where $\partial_{\bar{\kappa}}$ is a formal derivative in $\bar{\kappa}$. Furthermore, the conserved charge Q_ζ must be well-defined for all ζ , not just for $\varepsilon(t) = 1$ as in (3.10). To understand the potential complications arising when extending from Q_{∂_t} to general Q_ζ , we examine the difference in their integrands and find the remarkably simple relation

$$\sqrt{-g}(k_\zeta[h; g] - \varepsilon(t)k_{\partial_t}[h; g])|_{d\phi \wedge d\theta} = -\frac{1}{4}\sqrt{\frac{\alpha\gamma}{\Gamma}}\varepsilon'(t)(r + \bar{\kappa})h_{r\phi} \quad (3.11)$$

valid for all r . It follows, in particular, that the divergent part of the asymptotic r -expansion of $k_\zeta[h; g]$ matches the divergent part of $\varepsilon(t)k_{\partial_t}[h; g]$ where

$$\begin{aligned} \sqrt{-g}k_{\partial_t}[h; g]|_{d\phi \wedge d\theta} &= \frac{1}{4}\sqrt{\frac{\alpha}{\Gamma^3\gamma}}\left\{-k^2\gamma^2r^{-1}h_{tt} + 2k\gamma[k^2\gamma - \Gamma r\partial_r]h_{t\phi} \right. \\ &\quad \left. + 2\Gamma[\Gamma - k^2\gamma]r^2\partial_\phi h_{r\phi} + [2\Gamma^2 - k^2\Gamma\gamma - k^4\gamma^2]rh_{\phi\phi}\right\} + \mathcal{O}(r^0) \end{aligned} \quad (3.12)$$

Here we have used that $\partial_r h_{\phi\phi} = \mathcal{O}(r^{-2})$. Since the right-hand side of (3.11) may violate the Jacobi identity for the Dirac brackets of the conserved charges Q_ζ , we require that $h_{r\phi}$ is a total ϕ -derivative on the boundary

$$h_{r\phi}|_{\partial\Sigma} = \partial_\phi H_{r\phi} \quad (3.13)$$

where it is noted that total ϕ -derivatives can be ignored in the computation of the conserved charge Q_ζ . Under the constraints (3.1), (3.10) and (3.13), only perturbations h preserving them and only background metrics g which can be reached from the near-extremal geometry via a path of such perturbations are considered. This should ensure that Q_ξ and Q_ζ are well defined.

Taking the constraint (3.10) explicitly into account, we find that Q_ζ can be written as

$$Q_\zeta = \frac{1}{8\pi} \int_{\partial\Sigma} \sqrt{-g}\hat{k}_\zeta[h; g] \quad (3.14)$$

where

$$\sqrt{-g}\hat{k}_\zeta[h; g] = \frac{k^2\bar{\kappa}\varepsilon(t)}{2}\sqrt{\frac{\alpha\gamma^3}{\Gamma^3}}\left(\frac{h_{tt}}{r^2} - \frac{kh_{t\phi}}{r}\right)d\phi \wedge d\theta + \dots \quad (3.15)$$

The dots indicate that terms not contributing to the charge (3.14) have been omitted. The conserved charge Q_ζ is thus non-vanishing for general boundary conditions satisfying the constraints (3.1), (3.10) and (3.13). We also find that the integrand in (3.7) for the conserved charge Q_ξ is given by

$$\begin{aligned} \sqrt{-g}k_\xi[h; g] &= \frac{1}{4}\sqrt{\frac{\alpha}{\Gamma^3\gamma}}\left(\gamma\epsilon(\phi)\left[-\frac{k\gamma h_{tt}}{r^2} + \frac{2(k^2\gamma - \Gamma r\partial_r)h_{t\phi}}{r} - 2k\Gamma r\partial_\phi h_{r\phi} - k(\Gamma + k^2\gamma)h_{\phi\phi}\right] \right. \\ &\quad \left. + 2\Gamma\epsilon'(\phi)\left[k\gamma rh_{r\phi} - \frac{\Gamma\partial_\phi h_{t\phi}}{r}\right] + \epsilon''(\phi)\left[\frac{\Gamma^2 h_{t\phi}}{r}\right]\right)d\phi \wedge d\theta + \dots \end{aligned} \quad (3.16)$$

It is recalled that the near-horizon geometry of the extremal Kerr-Newman-AdS-dS black hole is obtained by setting $\bar{\kappa} = 0$ in which case the conserved charge Q_ζ is seen to vanish. The conserved charge Q_ξ , on the other hand, is unaffected by setting $\bar{\kappa} = 0$.

3.2 Central charges and entropy

Following the recipe used in [2], for example, we introduce the basis

$$\epsilon_n(\phi) = -e^{-in\phi} \quad (3.17)$$

and work out the corresponding Dirac bracket algebra (3.9). We thus find

$$i\{Q_{\xi_n}, Q_{\xi_m}\} = (n-m)Q_{\xi_{n+m}} + \frac{c_L}{12}n(n^2-1+b)\delta_{n+m,0} \quad (3.18)$$

where b is a constant while the central charge is given by

$$c_L = 3k \int_0^\pi \sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)} d\theta \quad (3.19)$$

After introducing the quantum generators

$$L_n = Q_{\xi_n} + \frac{bc_L}{24}\delta_{n,0} \quad (3.20)$$

and performing the usual substitution $\{.,.\} \rightarrow -i[.,.]$ of Dirac brackets by quantum commutators (recalling that we are using the unit convention $\hbar = 1$), the quantum charge algebra is recognized as the centrally-extended Virasoro algebra

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c_L}{12}n(n^2-1)\delta_{n+m,0} \quad (3.21)$$

With the expressions (2.4) and (2.5) appearing in the near-horizon geometry (2.21) of the near-extremal Kerr-Newman-AdS-dS black hole, the central charge is worked out to be

$$c_L = \frac{12r_+ \sqrt{\left(r_+^2 + \frac{3r_+^4}{\ell^2} - q^2\right)\left(1 - \frac{r_+^2}{\ell^2}\right)}}{1 + \frac{6r_+^2}{\ell^2} - \frac{3r_+^4}{\ell^4} - \frac{q^2}{\ell^2}} \quad (3.22)$$

This is the same expression as the one for the central charge characterizing the chiral CFT dual to the near-horizon geometry of the *extremal* Kerr-Newman-AdS-dS black hole evaluated in [16].

As in the case of the near-NHEK geometry considered in [21], we find that the central charge c_R associated with the conserved charges Q_ζ vanishes. By evaluating $\sqrt{-g}k_\xi[\mathcal{L}_\zeta g; g]$ and $\sqrt{-g}k_\zeta[\mathcal{L}_\xi g; g]$ explicitly, it is also verified that there is no central charge mixing the two conformal sectors characterized by the central charges c_L and c_R . This establishes that the two corresponding Virasoro algebras are mutually commutative and suggests that the CFT dual to the *near-extremal* Kerr-Newman-AdS-dS black hole is *non-chiral*. Since $c_R = 0$, the Cardy formula for the entropy of this dual CFT is simply given by

$$S = \frac{\pi^2}{3}c_L T_L = \frac{\pi(r_+^2 + a^2)}{\Xi} \quad (3.23)$$

This is readily recognized as the entropy (2.3) of the Kerr-Newman-AdS-dS black hole in the (near-)extremal limit.

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